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$x^2 + y^2 = ax$ is the equation of the cylinder.

For the surface of the paraboloid within the cylinder,

$$\sqrt{1 + (dy/dx)^2 + (dy/dz)^2} = \frac{\sqrt{a}}{2} \sqrt{\frac{a+4z}{az-x^2}}.$$

$$\therefore s = \sqrt{a} \int_0^a \int_z^{\sqrt{az}} \sqrt{\frac{a+4z}{az-x^2}} dz dx$$

$$= \frac{\sqrt{a}}{2} \int_0^a [\pi - 2\sin^{-1}\sqrt{z/a}] \sqrt{a+4z} dz = \frac{\sqrt{a}}{12} \int_0^a \frac{(a+4z)^{\frac{3}{2}} dz}{\sqrt{az-z^2}} - \frac{\pi}{12} a^2.$$

$$\text{Let } z = a \sin^2 \frac{1}{2} \theta.$$

$$\therefore s = \frac{a^2}{12} \int_0^\pi [1 + 4\sin^2 \frac{1}{2} \theta]^{\frac{3}{2}} d\theta - \frac{\pi}{12} a^2. \quad \text{Let } \theta = \pi - 2\varphi.$$

$$\therefore s = -\frac{a^2}{6} \int_0^{\frac{1}{2}\pi} (5 - 4\sin^2 \varphi)^{\frac{3}{2}} d\varphi - \frac{\pi}{12} a^2$$

$$= \frac{a^2}{18} [12E(2/\sqrt{5}, \frac{1}{2}\pi) - F(2/\sqrt{5}, \frac{1}{2}\pi)] - \frac{\pi}{12} a^2.$$

$\therefore s$ and $s_1 - s$ are the parts of the surface.

For the volume common to both.

$$v = 2 \int_0^a \left[\int_z^{\sqrt{az}} \sqrt{az-x^2} dx + \int_0^z \sqrt{ax-x^2} dx \right] dz$$

$$= \frac{1}{3} \int_0^a \left[\pi a^2 + \pi az - 4a\sqrt{az-z^2} - 8az\sin^{-1}\sqrt{z/a} + 2a^2\sin^{-1}\left(\frac{2z-a}{a}\right) \right] dz$$

$$= \frac{5}{32} \pi a^3. \quad v_1 - v = \frac{11}{32} \pi a^3.$$

Also solved by *L. C. WALKER*.

MECHANICS.

123. Proposed by *W. J. GREENSTREET, M. A.*, Editor of *The Mathematical Gazette*. Stroud, Gloucestershire, England.

Two equal uniform rods AB , BC are freely hinged at B ; C rests on rough horizontal plane, and A is attached to point above it. When C is as far as possible from A for equilibrium, AB , BC make angles α , β , respectively, with the vertical. Find the coefficient of friction between the rod at C and the plane.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $AB=BC=2a$, be the rods; CN , PF the verticals at C , A , respectively; μ =coefficient of friction at C ; R , the normal reaction at C ; S , T , the vertical and horizontal reaction at A ; W , the weight of each rod; $\angle NCB=\beta$, $\angle BAP=\alpha$.

Resolving vertically, $R+S=2W\dots(1)$.

Resolving horizontally, $\mu R=T\dots(2)$.

Taking moments about B with reference to the rod CB , $2aR(\sin\beta-\mu\cos\beta)=aW\sin\beta\dots(3)$.

Taking moments about B with reference to the rod AB , $2aS\sin\alpha-2aT\cos\alpha=aW\sin\alpha\dots(4)$.

(1) and (2) in (4) gives $2a(2W-R)\sin\alpha-2a\mu R\cos\alpha=aW\sin\alpha\dots(5)$.

Eliminating R between (3) and (5),

$$\frac{3\sin\alpha}{\sin\alpha+\mu\cos\alpha} = \frac{\sin\beta}{\sin\beta-\mu\cos\beta}.$$

$$\therefore \mu = \frac{2\sin\alpha\sin\beta}{3\sin\alpha\cos\beta + \cos\alpha\sin\beta} = \frac{2}{\cot\alpha + 3\cot\beta}.$$

124. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

At one end of a weightless thread of length l is fastened a sphere of radius r , and the other end of the thread is fastened to a vertical axis. The axis is put into motion of constant angular velocity ϕ . What is the maximum angle which the thread will make with the vertical axis?

Solution by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Let $PS=l$, $\angle SPO=\theta$, then $\tan\theta=\frac{mv^2}{r}/mg\dots(1)$, in which $v^2=\phi^2$, and $r=l\sin\theta$. $\therefore \tan\theta=\phi^2/lg\sin\theta\dots(2)$.

Since $\sin\theta=\tan\theta/\sqrt{1+\tan^2\theta}$, we have from (2) the radical equation

$$lg\tan^2\theta=\phi^2\sqrt{1+\tan^2\theta}\dots(3).$$

$$\therefore \theta=\tan^{-1}\left(\frac{\phi\sqrt{\{2[\phi^2\pm\sqrt{(\phi^2+4)}]\}}}{2lg}\right)\dots(4).$$

The *maximum* angle is made when $\phi=\infty$; and this angle is theoretically 90° . The *minimum* angle is made when $\phi=0$; and this angle is theoretically 0° . These extreme values of θ pre-suppose a *finite* value for l ; but if l be made zero, θ will be a maximum angle regardless of the magnitude of the angular velocity.

Scholium.—The principle of this problem is exemplified in the opening of an umbrella by twirling it quickly when held in the regular way. The engine-

